Structural Equation Models

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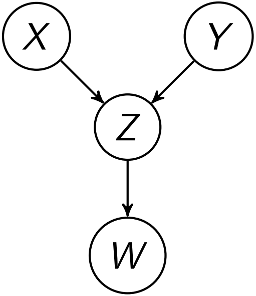
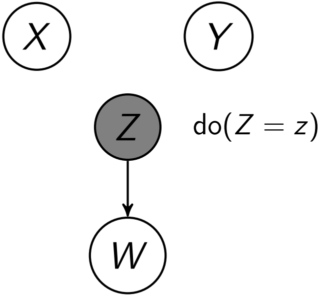
Structural Equation Models (SEMs)

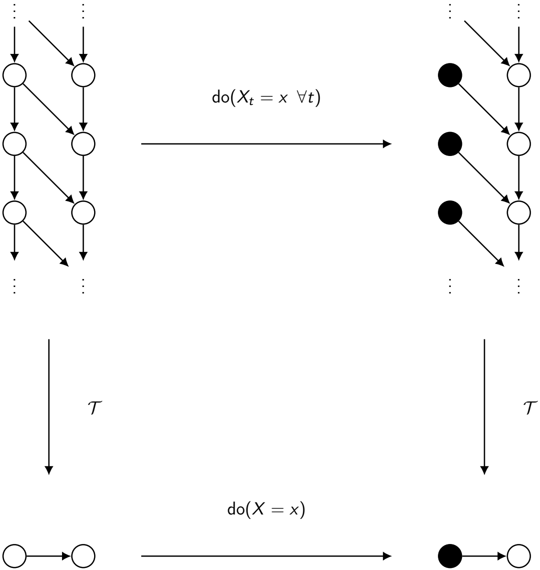




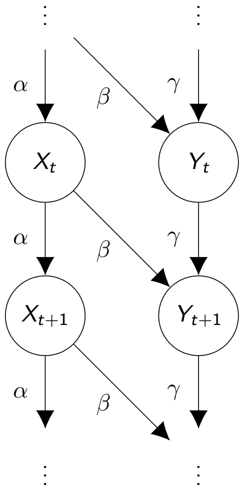
$$
\begin{aligned}
X &= f_X(E_X) \\
Y &= f_Y(E_Y) \\
Z &= f_Z(X, Y, E_Z) \\
W &= f_W(Z, E_W)
\end{aligned}
$$

$$
\begin{aligned}
X &= f_X(E_X) \\
Y &= f_Y(E_Y) \\
Z &= z \\
W &= f_W(Z, E_W)
\end{aligned}
$$

SEMs as abstractions



Example: Linear Gaussian Dynamics



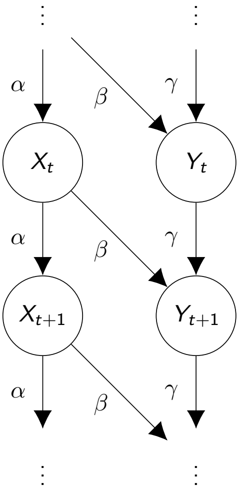
$$X_{t+1} = \alpha X_t + \mathcal{N}(0, \sigma_X^2)$$
$$Y_{t+1} = \beta X_t + \gamma Y_t + \mathcal{N}(0, \sigma_Y^2)$$

Suppose observations come from the stationary distribution

$$\implies \begin{cases} \mathbb{P}_{XY} \\ \mathbb{P}_{Y|\mathrm{do}(X=x)} \\ \mathbb{P}_{X|\mathrm{do}(Y=y)} \end{cases}$$

Problem: unless $\alpha = 1, \sigma_X^2 = 0$, cannot express distributions using unconfounded SEM.

Example: Linear Gaussian Dynamics



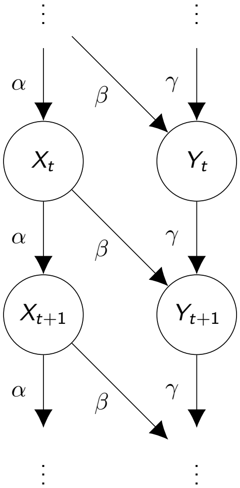
Define 'blurry measurements':

$$X^{(T)} = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} X_t$$

$$Y^{(T)} = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} Y_t$$

As $T \to \infty$, $(X^{(T)}, Y^{(T)}) \xrightarrow{d} (X, Y)$ for which unconfounded SEM can express observational and interventional distributions.

Example: Linear Gaussian Dynamics



Introduce approximation error:

$$\epsilon = \min_{SEM} \sup_{\zeta \in \{\text{interventions}\}} KL[\mathbb{P}_{\text{do}(\zeta)} | \hat{\mathbb{P}}_{\text{do}(\zeta)}]$$

where $\mathbb{P}_{\text{do}(\zeta)}$ is from true model, $\hat{\mathbb{P}}_{\text{do}(\zeta)}$ is from SEM approximation.

Then $\epsilon < \infty$, and $\epsilon \to 0$ as $\alpha \to 1$, $Var(X) \to c$